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LIFE TESTING

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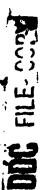
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1. INTRODUCTION

The term "life testing" is used to describe the designing of experiments to collect life-length data. Such data are used to estimate certain parameters which are commonly used by actuaries, biostatisticians, reliability engineers, and other life data analysts, or to make a decision as to whether to accept or to reject a lot (batch) of items. Examples of the parameters of interest are, the "mean time to failure", the "failure rate", the "survival function", the "reliable life", etc. Life testing is routinely undertaken in industrial environments such as the automobile, telecommunications

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and electronic industries, in military and defense-related industries, and in the biological- and health-related activities such as drug screening and bioassay experiments. In many instances, life testing is also undertaken to satisfy contractual and regulatory requirements.

Since the purpose of a life test is to obtain life length data, a random sample of items is taken, and the time to failure or time of withdrawal from test of each of these items is recorded. In a typical situation, the items are tested in an environment as nearly as possible identical to the environment in which the items are designed to operate. Such tests are called ordinary life tests. If the items in question have large times to failure, then an ordinary life test will involve an inordinate amount of test time. Thus, it is a common practice to subject the items to a test environment with greater than nominal stress. Such tests are called accelerated life tests, or overstress life tests; in some "military standards" documents, they are also referred to as environmental tests. Because of the fact that modern technology has been so successful in creating long life items, accelerated life tests are now more often undertaken than ordinary life tests. There are several strategies for performing accelerated life tests, and there are some special difficulties in making meaningful inferences from them. These will be described later.

With both ordinary or accelerated life tests, it is common that all the items under test will not be observed until failure. That is, some of the items will be withdrawn or removed from the life test. When this happens, the life test is said to be a censored-sample life test. A life test without any censoring is said to be a complete-sample life test. In industrial life testing, censoring is often undertaken to save on test time, or to save on the number of items that are tested until failure. In biological life testing, especially that involving



human subjects, censoring is due to causes which are beyond the control of the experimenter, such as the subject's being unwilling to continue, or because he or she leaves the area in which the test is taking place. Again, there are different strategies for censoring (when control is in the hands of the experimenter), each dictated by the situation at hand and each involving its own nuances. These will be described later.

A life test (ordinary or accelerated), in which the number of items to be tested is fixed in advance, is called a fixed-sample life test. A fixed-sample test could be either a complete-sample test or a censoredsample test, depending on whether there are any removals or withdrawals during the test. Fixed-sample tests are typically used when the goal of life testing is the estimation of unknown parameters. In contrast to fixed-sample tests are the sequential tests, for which the number of items to be tested is a random variable. Sequential tests are used when the goal of life testing is to make a decision as to whether a batch of items satisfies or fails to satisfy a specified life requirement, such as the mean time to failure being equal to or greater than a specified value. Sequential tests have the advantage that the expected number of items that are tested is smaller than that required in fixed-sample tests having the same performance characteristics. Lifetime data from a sequential life test can be used to estimate the parameters of interest mentioned earlier, but these estimates have to be used judiciously, since they do not have the same properties as estimates obtained from fixed samples.

2. CENSORING PATTERNS

As stated earlier, censoring is said to occur when unfailed items are withdrawn from the life test. In some instances, the experimenter has control over the amount and the times of censoring; in others, he



does not. Conceptually, the simplest form of censoring is that under the control of the experimenter which is known as Type-I censoring.

2.1 Type-I Censoring

Here, the life test is begun by observing the times to failure of the failed items among a predetermined number of items, say n, under an ordinary or an accelerated life test. The life test is terminated at a predetermined time, say t_0 , or at the time to failure of the last of the n items, whichever occurs first. The number of items which fail by time t_0 , say s, is a random variable, where $s \le n$, and the lifetime data consists of the s observable times to failure $X_{(1)} \le X_{(2)} \le \cdots \le X_{(s)}$, and the knowledge that (n-s) items have survived until time t_0 . The random variable $X_{(i)}$ denotes the i^{th} observed time to failure, $i=1,\cdots,s$, $s \le n$.

The main advantage of Type-I censoring is that the test procedure must terminate by time t_0 , so that the experimenter knows in advance the time at which the experiment will be completed. A disadvantage of Type-I censoring is the possibility of observing no failures by time t_0 . This poses some problems in obtaining the maximum-likelihood estimators of the mean-time-to-failure. There are, however, no difficulties in obtaining a Bayes estimator [Mann, Schafer, and Singpurwalla, pp. 399-404 (1975)]. Thus, in choosing t_0 , it is helpful if the experimenter has some prior knowledge about the failure behavior of the items.

Even though Type-I censoring is conceptually simple, a sample-theory (non-Bayesian) analysis of lifetime data from Type-I censored life tests poses several difficulties. These have been dealt with by Bartholomew (1963) and by Yang and Sirvanci (1977). A Bayesian analysis is straightforward provided that the user can specify meaningful priors for the parameters of interest.



2.2 Type-II Censoring

In Type-II censoring, the life test is begun by observing the times to failure of the items failing among a predetermined number of items, say n, under an ordinary or an accelerated life test. The life test is terminated at the time of failure of the rth item, where $1 \le r \le n$, is also predetermined. Thus, with Type-II censoring, the time at which the life test is terminated is a random variable. The lifetime data from this test consist of the observable times to failure $X_{(1)} \le X_{(2)} \le \cdots \le X_{(r)}$, together with the knowledge that (n-r) items have survived until time $X_{(r)}$. Since $r \le n$, $X_{(r)} \le X_{(n)}$. Thus, an advantage of Type-II censoring is the saving in test time and the fact that not all items tested are allowed to fail. The unfailed items could conceivably be put to some other use. A disadvantage of Type-II censoring is that the experimenter does not know in advance how long it will take to complete the test. Thus, the choice of r must be made on the basis of prior knowledge about the failure behavior of the device, so that the time to termination of the test, $X_{(r)}$, is well within the amount of time he has available for the life test. Methods for analysis of data from such tests were originally proposed by Epstein and Sobel, and are summarized in Mann, Schafer and Singpurwalla, pp. 163-174 (1975).

2.3 Combination of Type-I and Type-II Censoring

Another form of censoring is to combine the features of Type-I and Type-II censoring. Here, the test is terminated either at a predetermined time, t_0 , or at $X_{(r)}$, the time to failure of the r^{th} item, whichever occurs first; r is also predetermined. If t_0 is smaller than $X_{(r)}$, then s, the number of failures observed by time t_0 is a random variable, and if t_0 is larger than $X_{(r)}$, the time to termination of the test is a random variable. The lifetime data consist of the observed times to



failure, and knowledge of the fact that a certain number of items have survived beyond t_0 or $X_{(r)}$, as the case may be. Techniques for analyzing lifetime data from life tests which have both Type-I and Type-II censoring, and when the lifetimes have an exponential distribution, are given by Epstein (1954). Fertig and Mann (1980) give sampling plans in which the tables allow for Type-II censoring, but a maximum time on test is specified.

2.4 Progressive Censoring

Progressive censoring, which is also known as "multiple censoring" or "hypercensoring", involves elimination from further observation some (though not all) of the surviving items at various stages of a life test. Those items which are remaining after each stage of censoring are continued to be observed until failure, or a subsequent stage of censoring.

Suppose that the life test is begun by observing the times to failure of a predetermined number of items, say n, under an ordinary or an accelerated life test. Let r be the number of items which fail, and let censoring occur progressively in k stages at times $T_1 < T_2 < \cdots < T_k$. At the ith stage of censoring, r_i items are selected randomly from the survivors at time T_i , and are removed from further observation; thus,

$$n = r + \sum_{i=1}^{k} r_i.$$

In Type-I progressive censoring, the T_i 's are predetermined, so that the number of survivors at times $T_1 < T_2 < \cdots < T_k$ are random variables. The r_i 's are not random variables — they are chosen by the experimenter based on the number of survivors at times T_i , $i=1,\cdots,k$.

In Type-II progressive censoring, the number of survivors at each stage of censoring are fixed, so that the T_1 's coincide with the times of failure, and are thus random variables. Here again, the r_1 's are not



random variables — they are chosen by the experimenter at the commencement of the life test.

Techniques for analyzing data from progressively censored life tests are given by Cohen (1963) and by Mann (1969,1971).

2.5 Random Censoring

This type of censoring usually occurs when a life test is performed on biological subjects, particularly humans. Here, the amounts and the times of censoring are not under the control of the investigator. Random censoring refers to the elimination from further observation of a surviving item or items, at random points in time, τ_i , $i=1,\cdots,k$, where $k \leq n$, and n is the number of items under observation at the start of the life test.

The data from a randomly censored life test consist of the random variables Y_1, Y_2, \cdots, Y_n , where

$$Y_i = \min(\tau_i, X_i)$$
 , $i=1, \cdots, n$

and X_1, \dots, X_n are the failure times of the n items if they are allowed to operate freely without censoring. Associated with each Y_i is an indicator variable δ_i , where $\delta_i = 0$, if the ith item is censored, that is $Y_i = \tau_i$, and $\delta_i = 1$, otherwise, for i=1,...,n. It is common to assume that the τ_i 's, i=1,...,k, are a random sample drawn independently of the X_i 's, and have an unknown distribution which is different from the distribution of the X_i 's.

The model for random censorship was proposed by Efron (1967); methods for the analysis of data from randomly censored life tests are given by Breslow and Crowley (1974).



3. ACCELERATED LIFE TESTING

Accelerated life testing is performed to shorten the time period of a life test. This is achieved by inducing early failures in the items under test, by allowing them to operate in an environment which is more severe (accelerated) than the normal (use conditions or nominal) environment. A more severe environment can be created by increasing one or more of the stress levels which constitute the environment, to values which are greater than their usual values. In accelerated life testing, any one of the censoring patterns discussed in Section 2 can be used. The main problem with inference from accelerated life tests is that statements about the failure behavior of the items at use conditions environment have to be made using life length data from the more severe environments. One way of approaching this problem is to assume that the life distributions under the various environmental conditions come from the same family of distributions which is specified in advance, but that the parameters of the distributions change with the stresses according to a specified relationship with unknown parameters. Such relationships are known as time transformation functions, of which the "power law" is prominent (see Mann, Schafer and Singpurwalla, p. 425 (1974), Nelson (1975), Singpurwalla and Al-Khayaal (1977), and Mann (1978) for some recent contributions). When the unknown family of life distributions cannot be specified but the time transformation function can be specified, then methods for estimating the parameters of the time transformation function are given by Shaked, Zimmer and Ball (1979); a method for testing hypotheses on the unknown distributions is given by Sethuraman and Singpurwalla (1980). The techniques of the above two papers have been combined by Shaked and Singpurwalla (1980a) to give a unified semiparametric approach to accelerated life testing. A nonparametric approach for inference from accelerated life



tests when neither the time transformation function nor the family of failure distributions can be specified, is considered by Proschan and Singpurwalla (1979,1980).

There are several means by which accelerated environments can be obtained. All of these involve increasing the values of one or more of the stress levels which constitute the environment from their nominal values to more severe values. In choosing the higher values of the stress levels, care should be taken to ensure that the environment is not so severe as to introduce completely different failure modes. Also, testing under very severe environments will not give much information about the life behavior of the items under the severe environments, because all the items under test will fail instantly, implying a degenerate distribution (possibly at zero) for the failure times.

3.1 Fixed-Stress Accelerated Test

The simplest method, both physically and from the point of view of analysis and inference, of obtaining an accelerated environment for conducting a life test, is to increase the value of one or more of the stresses and hold them fixed over time. Such tests are known as fixed-stress tests. The accelerated life test experiment involves choosing several such increased values of the stress, and then performing either a complete-sample or a censored-sample life test at each of the increased values. For example, if the environment consists of a single stress, described by a variable, say V, and if V_0 denotes the use conditions stress, then it is common to consider k values of V, say $v_1 < v_2 < \cdots < v_k, \text{ for } k \ge 1, \text{ and perform a life test at each of the k values. Note that } v_0 < v_1. \text{ If the environment consists of two stresses, say V and H, with } v_0 \text{ and } H_0 \text{ being the use conditions stresses, then it is common to choose k values of V, } v_1 < v_2 < \cdots < v_k, \text{ and } \ell \text{ values of H,}$



 $H_1 \leq H_2 \leq \cdots \leq H_\ell$, and perform a total of m life tests, each at a combination of values of V and H, say (V_i, H_j) , $0 \leq i \leq k$, $0 \leq j \leq \ell$, and excluding, if necessary, the value (V_0, H_0) ; note that $m \leq (k+1)(\ell+1)$. The number of choices for performing accelerated life tests increases as the number of stresses which constitute the environment increases. All the references described above pertain to fixed-stress accelerated tests.

3.2 Step-Stress Accelerated Tests

Instead of choosing a high value of the stress and holding it fixed over all time, in step-stress testing, the higher values of the stress are introduced in stages, so that they form a step function over time. Specifically, consider the single stress environment of Section 3.1 with $V_0 < V_1 < \cdots < V_k$. Then, in step-stress testing, the life test is started off by choosing any value of V, say V_i , $0 \le i < k$, and observing the lifetimes of say n items under stress V_i , for τ_i units of time, where τ_i is preselected. At time τ_i , the stress level is increased to V_{i+j} , $1 \le j \le k-i$, and the lifetimes of the remaining items observed for an additional preselected τ_{i+j} units of time. At time $\tau_i + \tau_{i+j}$, the stress level is again increased to V_{i+j+p} , $1 \le p \le k-i-j$, and so on.

Even though step-stress testing is commonly undertaken in practice, there is a limited amount of literature dealing with the analysis of lifetime data from such tests. The papers by DeGroot and Groel (1979), Nelson (1980), and Shaked and Singpurwalla (1980b), address the various issues that arise when dealing with data from step-stress tests.

3.3 Accelerated Tests with Continuously Increasing Stresses

In step-stress testing, the various higher values of the stresses were introduced in stages, holding the values of the stresses fixed over



time within each stage. In tests with continuously increasing stresses, the stress increases over time according to some specified functional relationship. A simple version of such a test with a single stress environment may be to allow the stress to linearly increase over time, and observe the life lengths of the failed items. Even though such tests have been discussed in the literature, there do not appear to be any methods for analyzing data from such tests.

4. SEQUENTIAL LIFE TESTS

Sequential life tests for the exponential distribution are so frequently used by agencies of the U.S. Government, that the underlying procedures have been incorporated (as MIL-STD-781C) into a series of documents called the Military Standards. Military Standards are issued by the Department of Defense, and they specify operating procedures for quality control, life testing, methods of environmental testing, procedures for accepting or rejecting a batch (lot) of items, etc., which must be used by all the agencies of the Department of Defense. Sequential life testing is performed mainly for the purpose of making a decision on whether to accept or to reject a lot of items on the basis of their life length characteristics. As a by-product, data from a sequential test can be used for estimation purposes.

In sequential life testing, instead of starting the life test by subjecting all the n items to the test environment, only one item is subjected to test initially. If the item fails before a specified time, another item is taken from the lot, at random, and is subjected to the test environment. If the first item survives past the specified time, the test is stopped and the batch of items is accepted. If the total lifetime of the first and the second item is less than a specified value, then a third item is subjected to the test environment. Then, if the



total lifetime of the first item (which has failed) and the second item (which may be surviving) exceeds a specified amount, the test is stopped and the batch of items accepted. If the number of failed items during a specified amount of test time exceeds a specified number, the testing is stopped and the batch of items is rejected. This procedure is continued, the testing being performed one at a time, until a decision to either accept or to reject the batch is made. The key feature of sequential testing is to save on the number of items that are tested by making an early decision on acceptance or rejection if the lot is very good or very bad, and to do a prolonged amount of testing if the lot is neither one of these two extremes.

Sequential life testing for the exponential life distribution was first introduced by Epstein and Sobel (1955). Even though MIL-STD-781C clearly specifies that the prescribed sequential procedures apply only when the underlying life distribution is exponential, there does remain the possibility that these could be misused. In a recent paper, Harter and Moore (1976) point out the consequences of using the MIL-STD-781C procedures when the underlying life distribution is Weibull. Montagne and Singpurwalla (1980) generalize Harter and Moore's results, to the case in which the underlying life distribution has a monotone failure rate. Sequential life tests for situations in which there is some prior knowledge about the scale parameter of the exponential distribution have been considered by Schafer and Singpurwalla (1970). Sequential life test procedures for other life distributions such as the Weibull, or the gamma, are not available.

Once a sequential life test is terminated, with a decision being made as to whether to accept or to reject the batch, the observed life lengths can be used to estimate the parameters of the life distribution in question. Methods for doing this have been discussed by Bryant and Schmee (1979) and by Siegmund (1979).

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